Classroom Activity: Thinking Questions

1. Suppose you are told that the acceleration function of an object is a continuous function $a(t)$. Let's say you are given that $v(0) = 1$.

   True or **False**: You can find the position of the object at any time t.

2. Let $f(x) = \frac{1}{x^2}$, and $F(x)$ be an antiderivative of f with the property $F(1) = 1$.

   True or **False**. $F(-1) = 3$.

3. If f is an antiderivative of g, and g is an antiderivative of h, then

   (a) h is an antiderivative of f

   (b) h is the second derivative of f

   (c) h is the derivative of f''

4. **True** or **False**: An antiderivative of a sum of functions, $f + g$, is an antiderivative of f plus an antiderivative of g.

5. **True** or **False**: An antiderivative of a product of functions, fg, is an antiderivative of f times an antiderivative of g.

6. **True** or **False**. If f is continuous on the interval $[a, b]$, then $\int_a^b f(x)dx$ is a number.

7. Read the following four statements and choose the correct answer below.

   If f is continuous on the interval $[a, b]$, then:

   (i) $\int_a^b f(x)dx$ is the area bounded by the graph of f, the x-axis and the lines $x = a$ and $x = b$

   (ii) $\int_a^b f(x)dx$ is a number

   (iii) $\int_a^b f(x)dx$ is an antiderivative of $f(x)$

   (iv) $\int_a^b f(x)dx$ may not exist

   (a) (ii) only

   (b) (i) and (ii) only

   (c) (i) and (iii) only

   (d) (iv) only

8. Water is pouring out of a pipe at the rate of $f(t)$ gallons/minute. You collect the water that flows from the pipe between $t = 2$ and $t = 4$. The amount of water you collect can be represented by:

   (a) $\int_2^4 f(x)dx$

   (b) $f(4) - f(2)$

   (c) $(4-2)f(4)$

   (d) the average of $f(4)$ and $f(2)$ times the amount of time that elapsed

9. A sprinter practices by running various distances back and forth in a straight line in a gym. Her velocity at t seconds is given by the function $v(t)$. What does $\int_0^{60} |v(t)|dt$ represent?

   (a) The total distance the sprinter ran in one minute

   (b) The sprinter's average velocity in one minute

   (c) The sprinter's distance from the starting point after one minute

   (d) None of the above

10. Suppose f is a differentiable function. Then $\displaystyle\int_0^x f'(t)\,dt = f(x)$

    (a) Always

    (b) Sometimes

    (c) Never

    Justify your answer.

11. **True** or **False**. If $\int f(x)\,dx = \int g(x)\,dx$, then $f(x) = g(x)$.

12. **True** or **False**. If $f'(x) = g'(x)$, then $f(x) = g(x)$.

13. Suppose the function $f(t)$ is continuous and always positive. If G is an antiderivative of f, then we know that G:

    (a) is always positive.

    (b) is sometimes positive and sometimes negative.

    (c) is always increasing.

    (d) There is not enough information to conclude any of the above.

14. If f is continuous and $f(x) < 0$ for all $x \in [a, b]$, then $\int_a^b f(x)dx$

   (a) must be negative
   (b) might be 0
   (c) not enough information

15. **True** or **False**. If f is continuous on the interval $[a, b]$, $\frac{d}{dx}\left(\int_a^b f(x)dx\right) = f(x)$.

16. Below is the graph of a function f.
   Let $g(x) = \displaystyle\int_0^x f(t)\,dt$. Then for $0 < x < 2$, $g(x)$ is

   (a) increasing and concave up.
   (b) increasing and concave down.
   (c) decreasing and concave up.
   (d) decreasing and concave down.

17. Below is the graph of a function f.
   Let $g(x) = \displaystyle\int_0^x f(t)\,dt$. Then

   (a) $g(0) = 0$, $g'(0) = 0$ and $g'(2) = 0$
   (b) $g(0) = 0$, $g'(0) = 4$ and $g'(2) = 0$
   (c) $g(0) = 1$, $g'(0) = 0$ and $g'(2) = 1$
   (d) $g(0) = 0$, $g'(0) = 0$ and $g'(2) = 1$

18. You are traveling with velocity $v(t)$ that varies continuously over the interval $[a, b]$ and your position at time t is given by $s(t)$. Which of the following represent your average velocity for that time interval:

(I) $\dfrac{\int_a^b v(t)dt}{(b-a)}$

(II) $\dfrac{s(b)-s(a)}{b-a}$

(III) $v(c)$ for at least one c between a and b

(a) I, II, and III

(b) I only

(c) I and II only

19. The differentiation rule that helps us understand why the Substitution rule works is:

(a) The product rule.

(b) The chain rule.

(c) Both of the above.

20. One way to compute 1/2 area of the unit circle is to integrate $\displaystyle\int_{-1}^{1} \sqrt{1-x^2}\, dx$.

Let t be the angle between a radius of the circle and the x-axis. Then the area of the half circle is

(a) $\displaystyle\int_0^\pi -\sin t\, dt$

(b) $\displaystyle\int_0^\pi -\sin^2 t\, dt$

(c) $\displaystyle\int_\pi^0 -\sin^2 t\, dt$

(d) $\displaystyle\int_0^\pi -\cos t\, dt$

21. The area of a circular cell changes as a function of its radius, r, and its radius changes with time $r = g(t)$. If $\dfrac{dA}{dr} = f(r)$, then the total change in area, ΔA between $t = 0$ and $t = 1$ is

(a) $\Delta A = \displaystyle\int_{\pi(g(0))^2}^{\pi(g(1))^2} dA$

(b) $\Delta A = \displaystyle\int_{g(0)}^{g(1)} f(r)\,dr$

(c) $\Delta A = \displaystyle\int_0^1 f(g(t))g'(t)\,dt$

(d) all of the above

22. The radius, r, of a circular cell changes with time t. If $r(t) = \ln(t+2)$, which of the following represent the change in area, ΔA, of the cell that occurs between $t = 0$ and $t = 1$?

(a) $\Delta A = \pi(\ln 3)^2 - \pi(\ln 2)^2$

(b) $\Delta A = \displaystyle\int_{\ln 2}^{\ln 3} 2\pi r\,dr$

(c) $\Delta A = \displaystyle\int_0^1 2\pi \frac{\ln(t+2)}{t+2}\,dt$

(d) all of the above